Honey Coiling - A Study on the Gravitational Regime of Liquid Rope Coiling

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1 Introduction

We report on the coiling motion a falling stream of viscous liquid starts to exhibit after the impact on a rigid surface. This phenomenon is often observed in everyday life when pouring shampoo onto one's palm or dripping honey on a toast. The problem was formulated as a task of the *International Young Physicists' Tournament* 2013 (brief IYPT) as follows:

"A thin, downward flow of viscous liquid, such as honey, often turns itself into circular coils. Study and explain this phenomenon."

Experimentally it is easily found that over a broad range of frequencies there appear to be four regimes of coiling. In literature there has already been a great study on the geometry of the viscous jet [1] and on all four regimes, wherefore proportionalities for the characteristic coiling frequencies have been obtained using dimensional analysis [2, 3].
In this article we report our findings on the initial process the stream undergoes in order to start the coiling motion. Furthermore we present an analytical derivation of the frequency relation of the gravitational regime and its physical interpretation. Therefore we introduce the idea of bending torque for viscous liquid jets.

2 Theory

2.1 Regimes of Coiling

A falling stream of viscous liquid that emerges from a nozzle at a fixed height is being accelerated due to gravity as it falls towards the ground. Shortly after the impact the liquid jet turns itself into a stable circular motion that results in building up a coil of viscous liquid which eventually either collapses or gradually flows outwards. In literature the coiling section is named the *coil* and the falling stream of fluid above it is called the *tail*.
It is found that for different release heights there are four distinctively different types of coils that form, which divide the phenomenon into four regimes of coiling. These regimes are characterised by the forces which are dominant in the bending process of the liquid beam. Since they mainly depend on the net acceleration on each segment of the tail right before it is bent into the coil we can discuss the regimes by looking at the geometry of the tail.
As we know from the continuity equation of liquid flow, if the volume flow rate is kept constant as the liquid jet is being accelerated and thus increases its velocity, its cross-sectional area has to decrease. Therefore we can tell by the change in diameter of the tail how high the net acceleration relative to gravity is. It is relatively easy to imagine, that for rather small release heights the liquid jet undergoes little acceleration and thus its cross-section does not decrease by a lot since the tail is still moving at rather low velocities. Therefore viscous forces are dominant in this regime relative to gravitational and

inertial forces ($F_V >> F_G, F_I$). In literature this regime is called the *viscous* regime. As we increase the height the beam can accelerate more and becomes thinner. In this regime the filament undergoes the greatest change in velocity due to gravity. Therefore upon impact it is mainly the gravitational pull acting on the segment that has to overcome the viscous forces to bend the filament ($F_G \approx F_V >> F_I$). Hence it is called the *gravitational* regime. Finally, for relatively large release heights the filament reaches such high velocities that viscous friction forces cancel out the gravitational pull and the net acceleration goes towards zero. In this regime we mainly look at the momentum of the segment being bent into the coil and thus inertial forces govern the process ($F_I \approx F_V \approx F_G$). Therefore it is called the *inertial* regime. Furthermore, it is found that at the transition between the gravitational and inertial regime there seems to be an additional regime with chaotic behavior. Thus it is called the *inertia-gravitational* regime.
In this article we will mainly focus on the gravitational regime.

2.2 General Relation for Coiling Motion

In order to make a general prediction for the coiling frequency we will first look at the circular motion of the coil. Generally, the angular frequency of any circular motion is the tangential velocity divided by the trajectory's radius as denoted in (1). For this phenomenon we can say that the velocity of the circular motion is the velocity at which the tail is laid down on the surface, which has to be the same as just above the coil and can therefore be found using the continuity equation as in (2).

$$\omega = \frac{v}{R} \tag{1}$$

$$v = \frac{Q}{A} = \frac{Q}{\pi r^2} \tag{2}$$

Where ω is the angular velocity, v the tangential velocity, R the radius of the coil, Q the flow rate and r the radius of the cross-section of the tail. The coiling frequency f is related to the angular velocity by $2\pi f = \omega$. By combining (1) and (2) the following general relation for the coiling motion can be derived.

$$f \propto \frac{Q}{r^2 R} \tag{3}$$

As we will later see in our experimental results this relation holds for any release height and thus for every regime. However, when investigating the coil's radius' dependency on the release height, one finds it to be very nonlinear. In the viscous regime it increases strongly with height and then over the gravitational and inertial regimes it decreases asymptotically. In order to make specific predictions for the coiling frequency of the gravitational regime we have to study the bending process of the filament in greater detail.

2.3 The Bending Torque

When a solid rod is bent over a certain curvature, a torque has to be applied on both ends. This bending torque M can be written as

$$M = \frac{1}{R} Y I, \tag{4}$$

where R denotes the radius of curvature, Y Young's modulus or elastic modulus and I is the moment of inertia of the cross-sectional area of the rod.
In our case the radius of curvature over which the filament is bent is obviously the radius of the coil itself and the second moment of inertia of the area is the one of a circle of radius r of the tail just above the coil.

$$I = \frac{\pi}{4} r^4 \tag{5}$$

However, generally fluids do not have Young's moduli. Nevertheless, when a filament of sufficiently viscous liquid is deformed by stresses, viscoelastic properties of the fluid have to be considered. The viscoelasticity Y_V of such a liquid is proportional to the shear rate the liquid undergoes and of course the dynamic viscosity η.
In the process of bending the filament into the coil the rate of shearing can be described by the change of velocity over the radial distance by the following relation.

$$Y_V \propto \eta \frac{dv}{dR} = \eta\, \omega = \eta \frac{v}{R} \tag{6}$$

If we now put (5) and (6) into the general equation for the bending torque we can derive a relation for the bending torque of viscous liquids with circular cross-sections.

$$M_V \propto \frac{\pi}{4} \frac{\eta\, v\, r^4}{R^2} \tag{7}$$

2.4 The Gravitational Regime

In the gravitational regime the force of gravity is dominant in the process of bending. Therefore, in our theoretical approach we regard the force of gravity, that is acting on the section of the tail that is bent into the coil, to be the driving force that creates the bending torque.

We can say that the power needed for bending is provided by the gravitational pull on it. Including power losses due to viscous friction and heating a proportionality between the potential power stored in the segment by gravity P_G and the power used for bending P_B is formulated.

$$P_G \propto P_B \tag{8}$$

The relation in (8) can be represented using the beding torque and the force of gravity in the following way.

$$F_G \cdot v \propto M \cdot \omega \tag{9}$$

Here, v is the velocity the segment travels at and ω is the angular velocity of the coiling motion the tail is bent into. The length of the segment that is bent has to be the same as the cicumference of the coil, as illustrated in Fig.1. We can thereby determine its mass and by using the earlier derived relation (7) for the bending torque of viscous liquids we can obtain the following characteristic relation for the radius R of the coil in the gravitational regime.

$$R \propto \left(\frac{\nu\, Q}{g}\right)^{\frac{1}{4}} \tag{10}$$

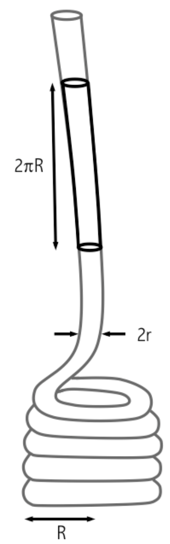

Figure 1: Schematic of tail above coil.

Here $\nu = \frac{\eta}{\rho}$ is the kinematic viscosity of the liquid. Now we can use this relation for the radius of the coil to derive the characteristic relation of the coiling frequency f_G in the gravitational regime by inserting it into the general relation for the coiling motion (3). Thereby, we arrive at this final expression.

$$f_G \propto \left(\frac{g\, Q^3}{\nu\, r^8}\right)^{\frac{1}{4}} \tag{11}$$

The characteristic coiling frequency of the other regimes can be found analogously by determining the dependency of the radius of the coil in each regime using the bending torque relation for viscous liquids.

3 Experimental Setup

The experiments were conducted using silicone oil of several viscosities, ranging from 5 $Pa \cdot s$ to 100 $Pa \cdot s$, since it is a newtonian fluid. The liquid was put into an injection upon which weights could be loaded to vary the flowrate out of the injection as can be seen in Fig.2. The falling height could be varied by changing the level of the container that was put onto a scale that was used to measure the flowrate. This was done taking the additional momentum of the tail into account. The phenomenon was filmed with a high-speed camera at up to 500 fps and then analyzed using the video analysis software *Logger Pro*.

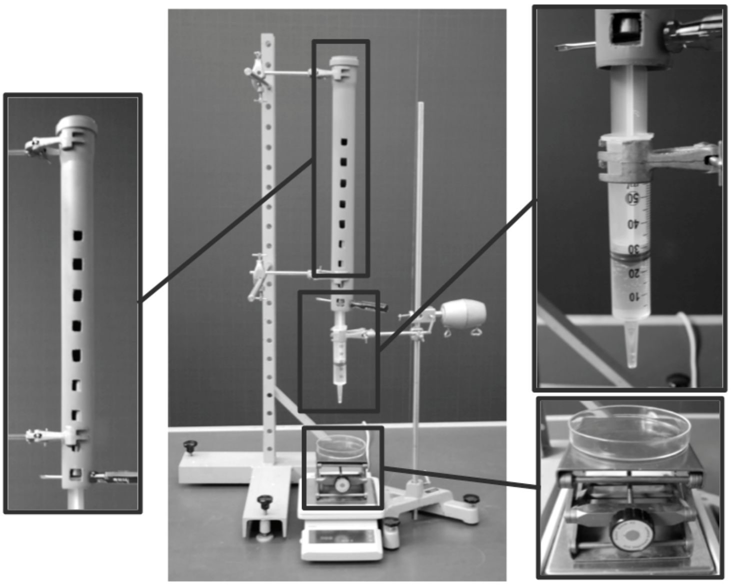

Figure 2: Experimental setup

4 Results and Discussion

In Fig.3 the initial process is split up into the main steps. As the beam falls towards the plate it accelerates (a). When it reaches the surface, it decelerates rapidly, however, due to its relatively high viscosity the fluid can not flow outwards and thus pressure is built up (b). As more liquid flows down that cannot flow away a pressure stasis within the beam is built up. Eventually the liquid filament undergoes self-buckling and bends outwards (c). The stream then falls outwards and the filament is layed onto the surface (d), until it reaches a point where viscous stresses inside the filament pull it backwards and it folds in on itself (e). The tail then starts an oscillatory motion and eventually turns itself into coiling as the circular motion can be regarded to be the optimal distribution of stresses.

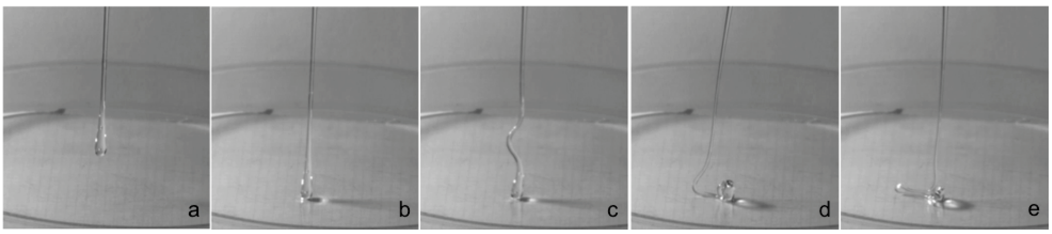

Figure 3: Initial process of liquid rope coiling

With the data obtained from our experiments we could verify our theoretical predictions of the coiling frequencies.

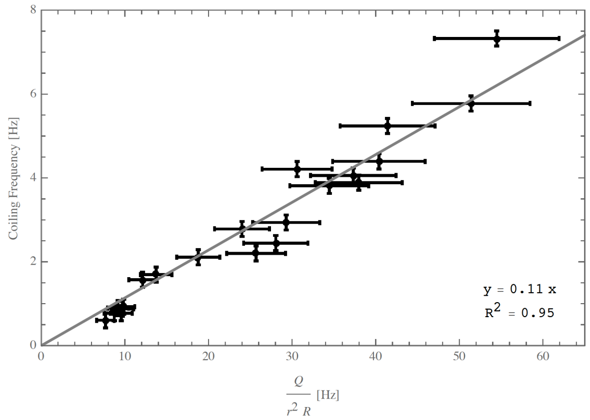

Figure 4: General coiling frequency relation.

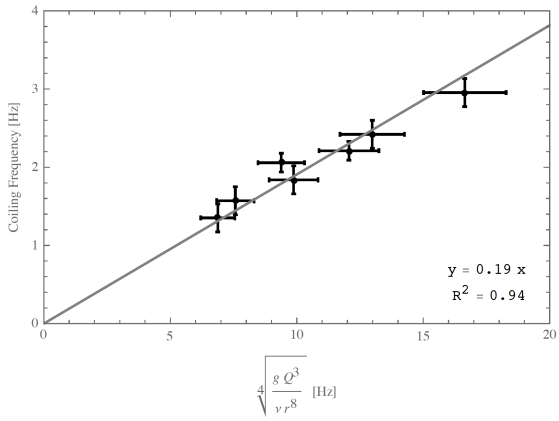

Figure 5: Coiling in gravitational regime.

As we can see in Fig.4 the linear relation that was predicted in (3) for the general coiling motion could be found experimentally. The errors arise mainly from the video analysis, where for very small scales the diameter of the tail could not be measured very accurately. Furthermore, the coiling frequency can be slightly perturbed as well, which is due to geometrical imperfections and the liquid below the coil flowing away. In Fig.5 the relation in (11) could be verified as well, where the errors arise for the same reason.

5 Conclusion

We have been able to explain the phenomenon's initial process that leads to the coiling motion. Furthermore, we have provided a way to derive the characteristic coiling frequency relations, which has so far only been possible using dimensional analysis.

6 Acknowledgement

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